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Charmonium and meson-molecule hybrid tetraquarks

Vector meson width and the isospin breaking in the X(3872) decay

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Abstract In the X(3872) decay, both of the $J/\psi\pi\pi$ and $J/\psi\pi\pi\pi$ branching fractions are observed experimentally, and their sizes are comparable to each other. In order to clarify the mechanism to cause such a large isospin violation, we investigate X(3872) employing a model of coupled-channel two-meson scattering with a $c\bar{c}$ core. The two-meson states consist of $D^0\bar{D}^{*0}$, D^+D^{*-} , $J/\psi\rho$, and $J/\psi\omega$. The effects of the ρ and ω meson width are also taken into account.

We calculate the transfer strength from the $c\bar{c}$ core to the final two-meson states. It is found that very narrow $J/\psi\rho$ and $J/\psi\omega$ peaks appear very close to the $D^0\bar{D}^{*0}$ threshold for a wide range of variation in the parameter sets. The size of the $J/\psi\rho$ peak is almost the same as that of $J/\psi\omega$, which is consistent with the experiments. The large width of the ρ meson makes the originally small isospin violation by about five times larger.

Keywords X(3872) · exotic hadrons · isospin symmetry breaking

1 Introduction

X(3872) has been found first by Belle [1] and then confirmed by various experiments [2, 3]. The mass of X(3872) is found to be 3871.57 ± 0.25 MeV, which is almost on the $D^0\bar{D}^{*0}$ threshold, 3871.73 MeV. The width is less than 2.3 MeV, which is very narrow for such a highly excited resonance, and $J^{PC}=1^{++}$ or 2^{-+} [2]. One of the peculiar features of X(3872) is that the isospin mixing occurs on a large scale. The decay branching fraction of X(3872) to three pions is comparable to that of two pions [4, 5]:

$$\frac{Br(X \rightarrow \pi^+\pi^-\pi^0 J/\psi)}{Br(X \rightarrow \pi^+\pi^- J/\psi)} = 1.0 \pm 0.4 \pm 0.3 \quad (\text{Belle}) \quad = 0.8 \pm 0.3 \quad (\text{BABAR}). \quad (1)$$

It has been pointed out in many works that X(3872) is not a simple $c\bar{c}$ state [2, 3]. The observed X(3872) mass is by about 78 MeV lower than the $J^{PC}=1^{++}$ $c\bar{c}$ state predicted by the quark model, which has successfully explained the $c\bar{c}$ mass spectrum below the $D\bar{D}$ threshold. As seen in Table 1, there are four thresholds which are very close to the X(3872) mass. It is natural to consider that X(3872) has large components of these two-meson states. The spectrum of the final pions, the radiative decay modes, the production rate, however, suggest that X(3872) has a $c\bar{c}$ core. We argue that X(3872) is a hybrid state of the $c\bar{c}$ core and the two-meson molecule with $J^{PC}=1^{++}$.

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Table 1 Mass and width of the mesons and relevant thresholds (in MeV). Data are taken from Ref.[2].

D^0	D^{*0}	J/ψ	ρ (Γ_ρ)	ω (Γ_ω)	D^+	D^{*+}	$D^0\bar{D}^{*0}$	$J/\psi\text{-}\rho$	$J/\psi\text{-}\omega$	D^+-D^{*+}
1864.80	2006.93	3096.92	775.49(149.1)	782.65(8.49)	1869.57	2010.22	3871.73	3872.41	3879.57	3879.79

Table 2 Model parameters for the interaction.

Parameter set	$V_{PP}^{(0)}$ (MeV·fm ³)	$V_{QP}^{(0)}$ (MeV·fm ^{3/2})	a (fm)	c	$E_0^{(Q)}$ (MeV)
A	3	10	0.4	1	3950
B	3.86	10	0.4	0.5	3950

2 Model Hamiltonian, the Lippmann-Schwinger equation, the transfer strength

The two-meson channels we consider are $D^0\bar{D}^{*0}$, D^+D^{*-} , $J/\psi\rho$, and $J/\psi\omega$. We introduce the orbitally excited $c\bar{c}$ core, which is treated as the bound state embedded in the continuum (BSEC) [6]. The two-meson states and the $c\bar{c}$ core are denoted by P -space and Q -space, respectively.

The model Hamiltonian is:

$$H = \begin{pmatrix} H_{PP} & V_{PQ} \\ V_{QP} & E_0^{(Q)} \end{pmatrix} = H_0 + V \quad \text{with} \quad H_0 = \begin{pmatrix} H_0^{(P)} & 0 \\ 0 & E_0^{(Q)} \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} V_{PP} & V_{PQ} \\ V_{QP} & 0 \end{pmatrix} \quad (2)$$

where H_{PP} is the Hamiltonian for the two-meson systems, V_{PQ} and V_{QP} are the transfer potentials and $E_0^{(Q)}$ is the BSEC mass before the coupling to the scattering states is switched on. We take its value from the prediction by the quark model.

Since the concerning particles are rather heavy and we consider the energy region very close to the threshold, we use a separable potential for the two-meson systems with the nonrelativistic treatment:

$$H_0^{(P)} = \sum_i \left(M_i + m_i + \frac{k_i^2}{2\mu_i} \right) \quad (3)$$

$$V_{PP;ij}(\mathbf{p}, \mathbf{p}') = V_{PP}^{(0)} \lambda_{ij} g(p)g(p')Y_{00}(\Omega_p)Y_{00}^*(\Omega_p') \quad \text{with} \quad g(p) = \exp\left[-\frac{a^2 p^2}{4}\right] \quad (4)$$

where M_i and m_i are the masses of the two mesons of the i -th channel, μ_i is their reduced mass, and k_i is their relative momentum. As for the range parameter, a , of the gaussian separable potential between the two mesons, V_{PP} , we use a common value for all the channels for simplicity. $V_{PP}^{(0)}$ is the strength of the interaction, and the factor λ_{ij} describes the channel dependence.

The transfer potentials between $c\bar{c}$ and the two-meson states, V_{PQ} are taken as:

$$\langle Q|V_{QP;i}|\mathbf{p}\rangle = V_{tr}^{(0)} \tilde{\lambda}_i g(p)Y_{00}^*(\Omega_p) \quad (5)$$

where $V_{tr}^{(0)}$ is the strength of the transfer potential, and the factor λ_i stands for the channel dependence.

The potential V is chosen not to violate the isospin symmetry. In addition, the direct coupling between $D^0\bar{D}^{*0}(c\bar{u}u\bar{c})$ and $D^+D^{*-}(c\bar{d}d\bar{c})$ is assumed not to occur because it is forbidden by the OZI rule. So, the V_{PP} for the $D\bar{D}^*$ isospin 1 system becomes the same as that of the isospin 0 system. We also assume that the $c\bar{c}$ core does not directly couple to $J/\psi\rho$ nor $J/\psi\omega$ because the transfer is forbidden again by the OZI rule. Moreover, the interaction between J/ψ and the vector meson is assumed to vanish, as it vanishes at the heavy quark mass limit. Thus, the channel dependence of the interaction is summarized as

$$\{\lambda_{ij}\} = \begin{pmatrix} -1 & 0 & c & c \\ 0 & -1 & -c & c \\ c & -c & 0 & 0 \\ c & c & 0 & 0 \end{pmatrix} \quad \text{and} \quad \{\tilde{\lambda}_i\} = (1 \ 1 \ 0 \ 0) \quad (6)$$

for the $D^0\bar{D}^{*0}$, D^+D^{*-} , $J/\psi\rho$, and $J/\psi\omega$ channels, respectively. The strengths of the potentials have been investigated but not well understood yet. Here we choose the values of $V_{PP}^{(0)}$ and $V_{QP}^{(0)}$ so that the X(3872) becomes a peak at the observed energy. The potential is considered to be short-ranged, so we

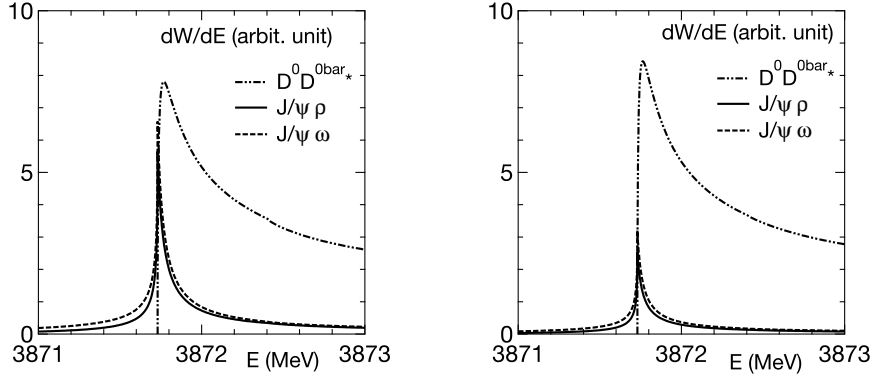


Fig. 1 The transfer strength from the $c\bar{c}$ core to the final states for the parameter set (a) A (b) B.

use a typical size, 0.4 fm, for the range parameter. As for the parameter c , we choose two different values are shown in Table 2.

We first solve the system without introducing the width of the vector mesons:

$$T = V + V G_0 T \quad \text{with} \quad T = \begin{pmatrix} T_{PP} & T_{PQ} \\ T_{QP} & T_{QQ} \end{pmatrix} \quad \text{and} \quad G_0 = \frac{1}{E - H_0 + i\epsilon}. \quad (7)$$

The ‘full’ propagator solved within the P -space, $G^{(P)}$, and the full propagator of Q , G_Q , can be obtained as

$$G^{(P)} = \left(E - H_0^{(P)} - V_{PP} + i\epsilon \right)^{-1} \quad \text{and} \quad G_Q = \left(E - E_0^{(Q)} - V_{QP} G^{(P)} V_{PQ} \right)^{-1} \quad (8)$$

It is considered that $X(3872)$ is produced via the $c\bar{c}$ state in the B meson decay. Thus, the observed mass spectrum is proportional to the transfer strength from the $c\bar{c}$ core to the final meson states:

$$\frac{1}{c_K} \frac{dW}{dE} = \sum_f \mu_f k_f |\langle f; k_f | T_{PQ} G_0 | c\bar{c} \rangle|^2 = -\frac{1}{\pi} \text{Im} \langle c\bar{c} | (T_{PQ} G_0)^\dagger G_0^{(P)} T_{PQ} G_0 | c\bar{c} \rangle \quad (9)$$

where c_K is a factor which comes from the kaon phase space, E is the energy of $D^0 \bar{D}^{*0}$ when the center of mass of the two mesons is at rest. The strength for the open channel f can be rewritten as

$$\frac{1}{c_K} \frac{dW(c\bar{c} \rightarrow f)}{dE} = \mu_f k_f \left| \langle f; k_f | (1 + V_{PP} G^{(P)}) V_{PQ} G_Q | c\bar{c} \rangle \right|^2. \quad (10)$$

Next we introduce the effects of the ρ and ω meson width. We assume that the decay of the vector mesons occurs only at the final two-meson states. Namely, we modify the free propagator within the P -space $G_0^{(P)}$ in eq. (9) as

$$G_0^{(P)} \rightarrow \tilde{G}_0^{(P)} = \left\{ E - (M_i + m_i + \frac{k^2}{2\mu_i}) + \frac{i}{2} \Gamma_V(s(k)) \right\}^{-1} \quad (11)$$

The width of the vector mesons, $\Gamma_V(s)$, depends on the energy of the pion relative motion, s , which depends on k . The parameters in Γ_V are taken so that it produces the observed ρ or ω width.

Thus we have the strength for the open channel f as

$$\frac{1}{c_K} \frac{dW(c\bar{c} \rightarrow f)}{dE} = \frac{2}{\pi} \mu_f \int \frac{k^2 dk \mu_f \Gamma_V(s(k))}{(k_f^2 - k^2)^2 + (\mu_f \Gamma_V(s(k)))^2} \left| \langle f; k | (1 + V_{PP} G^{(P)}) V_{PQ} G_Q | c\bar{c} \rangle \right|^2. \quad (12)$$

The detailed calculation is given in ref. [6].

3 Results

In Fig. 1, we show the transfer strength from the $c\bar{c}$ core to the final two-meson states with the parameter set A or B. In both of the cases, the strengths of $J/\psi\rho$ and $J/\psi\omega$ make a very thin peak

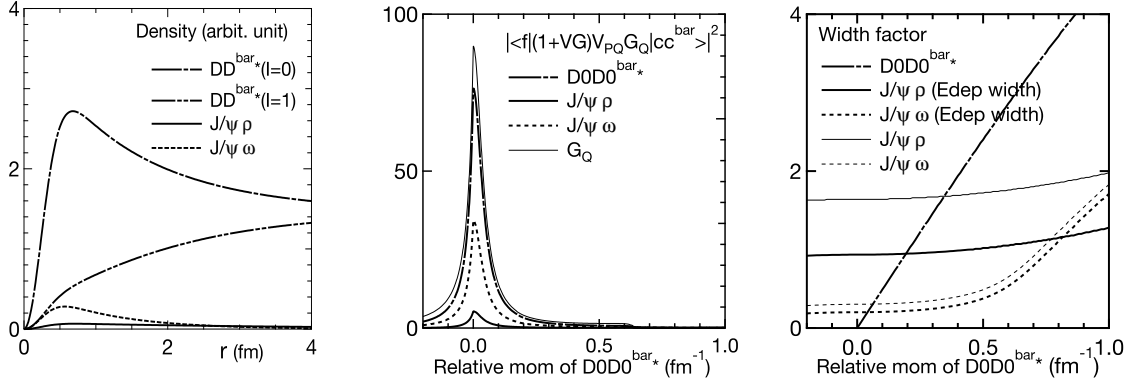


Fig. 2 (a) The density of the resonance state, (b) the matrix elements for the transition (see text), and (c) the factors from the meson width.

at the $D^0\bar{D}^{*0}$ threshold, and their sizes are comparable to each other. Though the model parameters we take are mostly empirical, it is found this feature appears for various kinds of model parameters, provided that the strength of $D^0\bar{D}^{*0}$ gathers closely above the threshold.

In Fig. 2(a), we show the density of the state at the $D^0\bar{D}^{*0}$ peak energy, 0.05 MeV above the threshold, for the parameter set A. The isospin 1 component of DD^{*0} is small at the short distance but becomes the same amount as that of the isospin 0 component at $r \rightarrow \infty$, because only the $D^0\bar{D}^{*0}$ channel is open at this energy.

In Fig. 2 (b), we plot $|\langle c\bar{c}|G_Q|c\bar{c}\rangle|^2$ and other matrix elements for the transition from $c\bar{c}$ to the final two-meson channels, $|\langle f|g^{-1}(1+V_{PP}G^{(P)})V_{PQ}G_Q|c\bar{c}\rangle|^2$. In Fig. 2(c), we show the factor from the meson width in the final states weighted by the interaction range: $\int \frac{\mu_f \Gamma_V(s(k))}{(k_f^2 - k^2)^2 + (\mu_f \Gamma_V(s(k)))^2} g(k)^2 k^2 dk$. This factor for $D^0\bar{D}^{*0}$ becomes linear because it becomes $k_f g(k_f)^2$ at $\Gamma_V \rightarrow 0$. As for the vector mesons, the factor is roughly proportional to the width Γ_V in the nominator. It is found that the peak feature of X(372) comes from the pole in G_Q . The isospin 1 component originates from the threshold difference in the propagator $G^{(P)}$, which is enhanced by the broad width Γ_ρ ; the factor for the ρ meson is by about five times larger than that of the ω meson.

4 Summary

We solve the Lippmann-Schwinger equation for the coupled-channel two-meson scattering problem ($D^0\bar{D}^{*0}$, $D^+\bar{D}^{*-}$, $J/\psi \rho$, and $J/\psi \omega$) with the $c\bar{c}$ core to investigate the features of X(372). The isospin breaking in the present model comes from the difference in the meson masses. The effects of the vector meson widths are also taken into account.

It is found that the transfer strength from the $c\bar{c}$ core to each of the $J/\psi \rho$ or $J/\psi \omega$ has a peak on the $D^0\bar{D}^{*0}$ threshold. The large width of the ρ meson enhances the isospin 1 component. The size of the $J/\psi \rho$ peak is almost the same as that of the $J/\psi \omega$ peak, which is consistent with the observed feature.

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